

Mathematical Formulations of Drought Indices: A Reference Compendium of Meteorological, Agricultural, Hydrological, and Remote-Sensing Indicators

Abstract

Drought is a complex hazard that manifests across meteorological, agricultural, hydrological, and vegetative domains, and its quantification has historically relied on a wide range of indices developed over the past six decades. This compendium assembles, in one reference, the complete mathematical formulation of the drought indices most widely used in operational drought monitoring: the Standardized Precipitation Index, the Deciles Index, the Percent of Normal Index, the China-Z Index and its modified variant, the Z-Score Index, the Effective Drought Index, the Standardized Precipitation-Evapotranspiration Index, the Reconnaissance Drought Index, the Palmer Drought Severity Index and its hydrological and self-calibrated variants, the Agricultural Reconnaissance Index, the Soil Moisture Deficit Index, the Evapotranspiration Deficit Index, the Keetch-Byram Drought Index, the Streamflow Drought Index, the Surface Water Supply Index, and three satellite-derived vegetation indices (VHI, NMDI, VSDI). Complementary statistical techniques for characterizing drought trends, periodicities, event severity, frequency, and large-scale climatic teleconnections are also presented. Each index is described with its full derivation, the physical meaning and units of every variable, and the classification scale conventionally used to interpret it, together with the primary literature from which each method originates.

1. Introduction

Drought has no single, universally accepted definition, and consequently no single index captures its full expression across the hydrological cycle. Meteorological drought reflects a deficit in precipitation relative to a long-term normal; agricultural drought reflects insufficient soil moisture to sustain crop and vegetation demand; hydrological drought reflects below-normal streamflow, reservoir storage, or groundwater levels; and vegetation-based (remote-sensing) drought indicators infer plant water stress from satellite-observed reflectance and land-surface temperature (Wilhite & Glantz, 1985; Mishra & Singh, 2010).

This compendium is organized by drought domain. Section 2 presents meteorological indices, beginning with the Standardized Precipitation Index — the World Meteorological Organization's recommended index for characterizing meteorological drought (WMO, 2012) — and continuing through the deciles-, anomaly-, and distribution-based indices commonly used alongside it, before turning to the evapotranspiration-informed indices (SPEI, RDI) and the Palmer family of soil-moisture-balance indices. Section 3 presents agricultural drought indices grounded in soil moisture and evapotranspiration deficits. Section 4 presents hydrological (streamflow-based) indices. Section 5 presents satellite-derived vegetation health indices. Section 6 summarizes the unit conventions traditionally associated with several of these methods. Section 7 presents complementary statistical techniques — trend detection, wavelet spectral analysis, drought-event severity extraction, severity-duration-frequency analysis, and teleconnection analysis — that are commonly applied to any of the indices above once computed. Section 8 collects the drought-severity classification scales conventionally associated with each index. Full bibliographic references follow in Section 10.

2. Meteorological Drought Indices

Meteorological drought indices are derived solely from precipitation records (and, for two indices in this section, potential evapotranspiration), making them the most widely applicable class of drought indicator, since precipitation is the most consistently observed climate variable worldwide.

2.1 Standardized Precipitation Index (SPI)

The Standardized Precipitation Index (McKee, Doesken, & Kleist, 1993, 1995) expresses observed precipitation, accumulated over a chosen time scale, as the number of standard deviations by which it departs from the long-term mean, after first normalizing the (typically right-skewed) precipitation distribution through a fitted probability distribution. It is computed by fitting a two-parameter gamma distribution to the accumulated precipitation series.

$$g(x) = [1 / (\beta^\alpha \Gamma(\alpha))] \times x^{\alpha-1} \times e^{(-x/\beta)}, \quad x > 0$$

where x is accumulated precipitation (mm, $x > 0$), $\alpha (> 0)$ is the distribution's shape parameter, $\beta (> 0)$ is its scale parameter, and Γ denotes the gamma function. The parameters are estimated by Thom's (1958) approximate maximum-likelihood method, as adopted by Edwards and McKee (1997):

$$A = \ln(\bar{x}) - (1/n) \sum \ln(x_i)$$

$$\alpha = (1 / 4A) \times (1 + \sqrt{1 + 4A/3}) \quad \beta = \bar{x} / \alpha$$

with $\bar{x}$ the mean and n the number of non-zero precipitation observations in the fitting period. Because a precipitation record may contain zero values, for which the gamma density is undefined, the cumulative probability is adjusted for the empirical probability of zero precipitation, q :

$$q = (N - n) / N \quad H(x) = q + (1 - q) \times G(x)$$

where N is the total number of observations and $G(x)$ is the fitted gamma cumulative distribution function. Finally, the cumulative probability $H(x)$ is transformed to the standard normal deviate that defines the index itself:

$$SPI = \Phi^{-1}(H)$$

where Φ^{-1} denotes the inverse cumulative distribution function of the standard normal distribution. SPI values are computed independently for each of several time scales — most commonly 1, 3, 6, 12, 24, and 48 months — allowing meteorological drought to be characterized from short-term (agricultural-relevant) to long-term (hydrological-relevant) accumulation periods, and the resulting values conventionally range between approximately +2.0 and -2.0.

2.2 Rainfall Anomaly Index (RAI)

The Rainfall Anomaly Index (van Rooy, 1965) ranks the precipitation record and separately averages its extremes to construct a bounded anomaly scale that does not require distributional fitting.

$$RAI = +3 \times (p - \bar{p}) / (\bar{m}_{high} - \bar{p}) \quad \text{if } p \geq \bar{p}$$

$$RAI = -3 \times (p - \bar{p}) / (\bar{m}_{low} - \bar{p}) \quad \text{if } p < \bar{p}$$

where p is the precipitation observed in the period being evaluated (mm), $\bar{p}$ is the long-term mean precipitation for that period, $\bar{m}_{high}$ is the mean of the ten highest values on record (used for positive anomalies), and $\bar{m}_{low}$ is the mean of the ten lowest values on record (used for negative anomalies).

2.3 Percent of Normal Precipitation Index (PN)

The Percent of Normal Index (Willeke, Hosking, Wallis, & Guttman, 1994) is among the simplest drought metrics, expressing observed precipitation as a percentage of its long-term average for the same period, and is applicable across monthly, seasonal, or annual accumulation periods.

$$PN = (P_i / \bar{P}) \times 100$$

where P_i is the precipitation observed in period i (mm) and $\bar{P}$ is the mean normal precipitation for that period across the full climatological record (mm).

2.4 Deciles Index (DI)

The Deciles Index (Gibbs & Maher, 1967) places each observation in its historical context by dividing the long-term, rank-ordered precipitation record for a given period into ten equal-frequency classes, or deciles.

$$DEC_n = \text{Percentile}(P_m, n) \quad n = 10, 20, \dots, 100$$

$$DI = n \quad \text{if} \quad DEC_{(n-1)} < P \leq DEC_n$$

where P_m is the full historical series of precipitation for calendar period m , and DEC_n is the value below which n percent of that historical record falls. Decile 1 represents the driest tenth of the historical record and decile 10 the wettest.

2.5 China-Z Index (CZI) and 2.6 Modified China-Z Index (MCZI)

The China-Z Index was introduced by the National Climate Center of China in 1995 as an alternative to the Standardized Precipitation Index for precipitation series better described by a Pearson Type III distribution than a gamma distribution (Wu, Hayes, Weiss, & Hu, 2001). It is derived from the Wilson-Hilferty cube-root transformation, which relates a chi-squared variable to a standard normal deviate.

$$\varphi = (X - \bar{X}) / \sigma$$

$$\sigma = \sqrt{[(1/n) \sum (X - \bar{X})^2]}$$

$$Cs = [\sum (X - \bar{X})^3] / (n \times \sigma^3)$$

$$CZI = (6/Cs) \times [(Cs/2) \times \varphi + 1]^{(1/3)} - 6/Cs + Cs/6$$

where X is precipitation for the period of interest (mm), $\bar{X}$ its long-term mean, σ its standard deviation, and Cs its coefficient of skewness. The Modified CZI (Wu et al., 2001) applies the identical transformation but substitutes the long-term median precipitation for the mean in the standardization step, a substitution intended to reduce sensitivity to outliers in strongly skewed climates.

2.7 Z-Score Index (ZSI)

The Z-Score Index standardizes precipitation using only its sample mean and standard deviation, with no correction for skewness. Where CZI and MCZI incorporate the sample coefficient of skewness through a Pearson Type III-based (Wilson-Hilferty) transformation, ZSI implicitly assumes the underlying precipitation distribution is symmetric, making it a simpler but less distributionally-informed alternative for markedly skewed precipitation records.

$$ZSI = (P_i - \bar{P}) / SD$$

where P_i is precipitation in the period of interest (mm), $\bar{P}$ the long-term mean precipitation (mm), and SD the standard deviation of precipitation across the record.

2.8 Effective Drought Index (EDI)

The Effective Drought Index (Byun & Wilhite, 1999) was developed to overcome a limitation common to monthly indices such as SPI — namely, that they treat each accumulation period independently and can therefore under-represent drought conditions that develop gradually across period boundaries. EDI instead accumulates a daily, exponentially-weighted measure of “effective precipitation” that explicitly accounts for water carried over from preceding days.

$$EP_i = \sum_{n=1}^{\infty} (DS)^n \left[\left(\sum_{m=1}^{\infty} (n)^m P_{(i-m)} \right) / n \right]$$

$$DEP_i = EP_i - MEP$$

$$EDI_i = DEP_i / SD(DEP)$$

where P is daily precipitation (mm), DS is the summation window in days (conventionally 365, or 15 for short-term applications), EP_i is the effective precipitation on day i, MEP is its long-term mean, and SD(DEP) is the standard deviation of the departure series. EDI is computed at daily resolution and conventionally ranges from about -2.5 to 2.5, with values between -1 and 1 read as near-normal and values below -2 read as severe drought.

2.9 Standardized Precipitation-Evapotranspiration Index (SPEI)

The Standardized Precipitation-Evapotranspiration Index (Vicente-Serrano, Beguería, & López-Moreno, 2010) extends the SPI methodology to incorporate atmospheric evaporative demand, giving it sensitivity to temperature-driven drought stress that SPI, by construction, cannot capture. It is based on the accumulated monthly climatic water balance:

$$D_i = P_i - ETo_i$$

where P_i is precipitation and ETo_i is reference evapotranspiration for the same period. The water-balance series D is fitted to a three-parameter log-logistic distribution using probability-weighted moments (Hosking, 1990):

$$f(x) = (\beta/\alpha) \times [(x-\gamma)/\alpha]^{\beta-1} \times [1 + ((x-\gamma)/\alpha)^{\beta}]^{-2}$$

$$\beta = (2w_1 - w_0) / (6w_1 - w_0 - 6w_2)$$

$$\alpha = (w_0 - 2w_1)\beta / [\Gamma(1+1/\beta) \Gamma(1-1/\beta)]$$

$$\gamma = w_0 - \alpha \Gamma(1+1/\beta) \Gamma(1-1/\beta)$$

where α , β , and γ are the scale, shape, and origin parameters of the fitted distribution, and w_0 , w_1 , w_2 are the first three probability-weighted moments of D. The cumulative probability is then obtained from the fitted distribution,

$$F(x) = [1 + (\alpha/(x-\gamma))^{\beta}]^{-1}$$

and standardized following the classical rational approximation to the inverse normal distribution (Abramowitz & Stegun, 1965):

$$W = \sqrt{-2 \ln P}$$

$$SPEI = \pm [W - (c_0 + c_1 W + c_2 W^2) / (1 + d_1 W + d_2 W^2 + d_3 W^3)]$$

where P is the probability of exceeding a given value of D , and $c_0=2.515517$, $c_1=0.802853$, $c_2=0.010328$, $d_1=1.432788$, $d_2=0.189269$, $d_3=0.001308$ are fixed coefficients of the approximation, taken with a positive sign when $P \leq 0.5$ and a negative sign otherwise.

2.10 Reconnaissance Drought Index (RDI)

The Reconnaissance Drought Index (Tsakiris & Vangelis, 2005) compares cumulative precipitation against cumulative potential evapotranspiration, giving it, like SPEI, sensitivity to evaporative demand.

$$\alpha_k = \sum_{i=Jan}^{Dec} P_i / \sum_{i=Jan}^{Dec} PET_i$$

RDI is expressed in either of two forms: a normalized form,

$$RDI_{normalized} = (\alpha_k / \bar{\alpha}) - 1$$

or a standardized form,

$$RDI_{standardized} = (\alpha_k - \bar{\alpha}) / \sigma$$

where $\bar{\alpha}$ is the mean of α across all years of record and σ its standard deviation.

2.11 Palmer Drought Severity Index (PDSI), Palmer Hydrological Drought Index (PHDI), and the Self-Calibrated PDSI

The Palmer Drought Severity Index (Palmer, 1965) remains the most widely known drought index in North American climatology, and is unique among the indices in this section in modeling a full soil-moisture water balance rather than standardizing precipitation alone. The formulation below follows the widely adopted computational procedure of Alley (1984), as implemented in the reference tool of Jacobi et al. (2013).

The water balance is computed for a two-layer soil model, comprising a shallow surface layer holding a fixed one inch of available water and an underlying layer holding the remainder of the soil's available water capacity (AWC):

$$PR = AWC - S_0 \quad (\text{potential recharge})$$

$$PRO = AWC - PR \quad (\text{potential runoff})$$

$$PL = PL_s + PL_u \quad (\text{potential loss, surface plus underlying layers})$$

where S_0 is the combined soil moisture at the start of the period. From the water balance, four climatic coefficients are estimated for each calendar month m from a long-term calibration period,

$$\alpha_m = ET_m^- / \hat{P} \hat{E} T_m^-,$$

$$\beta_m = R_m^- / P \bar{R}_m,$$

$$\gamma_m = R \bar{O}_m / P \bar{R} O_m,$$

$$\delta_m = \bar{L}_m / \bar{P} L_m$$

and used to compute the precipitation that would be climatically appropriate for existing conditions (CAFEC), and the departure of actual precipitation from it:

$$CAFEC = \alpha \cdot PET + \beta \cdot PR + \gamma \cdot PRO - \delta \cdot PL \quad d = P - CAFEC$$

A regional weighting factor K , larger in arid climates and smaller in humid ones, converts the moisture departure into the monthly moisture anomaly index Z (Palmer, 1965; Alley, 1984):

$$K'_m = 1.5 \times \log_{10} \left[\left(\hat{T}_m + 2.8 \right) / \hat{D}_m \right] + 0.5$$

$$K_m = 17.67 \times K'_m / \Sigma_m (\hat{D}_m \times K'_m) \quad Z = K \times d$$

Finally, PDSI is obtained from Z through a backtracking recursion that determines, retrospectively, when a wet or dry spell became established and how it should be scored throughout its duration:

$$X1 = \max(0, 0.897 \times X1_{(prev)} + Z/3)$$

$$X2 = \min(0, 0.897 \times X2_{(prev)} + Z/3)$$

with a wet spell considered established once $X1$ reaches 1, and a drought considered established once $X2$ reaches -1 . The Palmer Hydrological Drought Index (PHDI) is derived from the same recursion, differing in the criterion used to determine when a spell has ended, and is generally regarded as more conservative — that is, slower to signal the end of a drought — than PDSI, making it more representative of hydrological (rather than purely meteorological) drought recovery. The Self-Calibrated PDSI (Wells, Goddard, & Hayes, 2004) follows the identical procedure but derives the climatic coefficients and the K weighting factor from each location's own record rather than from Palmer's original fixed empirical constants, improving spatial comparability between climatically dissimilar regions.

3. Agricultural Drought Indices

Agricultural drought indices are grounded in soil-moisture and evapotranspiration deficits, making them more directly relevant to crop and vegetation stress than the precipitation-only indices of Section 2.

3.1 Agricultural Reconnaissance Index (ARI)

The Agricultural Reconnaissance Index expresses monthly precipitation relative to reference evapotranspiration as a simple water-balance ratio (Nieuwolt, 1981), providing a rapid indicator of agricultural water adequacy.

$$ARI = (P / ETo) \times 100$$

where P is monthly precipitation (mm/month) and ETo is monthly reference evapotranspiration (mm/month).

3.2 Soil Moisture Deficit Index (SMDI)

The Soil Moisture Deficit Index (Narasimhan & Srinivasan, 2005) characterizes agricultural drought from the departure of root-zone soil water from its long-term normal, using the historical median rather than the mean as the reference — a choice motivated by the median's greater resistance to distortion by outlier years.

$$SD = (SW - MSW) / (MSW - minSW) \times 100 \quad \text{if } SW \leq MSW$$

$$SD = (SW - MSW) / (maxSW - MSW) \times 100 \quad \text{if } SW > MSW$$

$$SMDI = 0.5 \times SMDI_{(prev)} + SD / 50$$

where SW is the soil water available in the root zone for the period being evaluated (mm), and MSW, minSW, and maxSW are the long-term median, minimum, and maximum soil water for that same calendar period. The weekly or monthly deficit SD ranges from -100 (severe deficit) to +100 (surplus), and the cumulative SMDI is scaled, following Palmer's (1965) precedent, to range approximately -4 to +4 for direct visual comparison with PDSI.

3.3 Evapotranspiration Deficit Index (ETDI)

The Evapotranspiration Deficit Index (Narasimhan & Srinivasan, 2005) applies the same anomaly-scaling principle as SMDI, but to a water-stress ratio derived from potential and actual evapotranspiration rather than to soil water directly.

$$WS = (PET - AET) / PET$$

$$WSA = (MWS - WS) / (MWS - minWS) \times 100 \quad \text{if } WS \leq MWS$$

$$WSA = (MWS - WS) / (maxWS - MWS) \times 100 \quad \text{if } WS > MWS$$

$$ETDI = 0.5 \times ETDI_{(prev)} + WSA / 50$$

where PET and AET are potential and actual evapotranspiration for the period, WS is the resulting water-stress ratio (0 indicating no stress, 1 indicating maximum stress), and MWS, minWS, and maxWS are its long-term median, minimum, and maximum for the corresponding calendar period. As with SMDI, ETDI is scaled to range approximately -4 to +4.

3.4 Keetch-Byram Drought Index (KBDI)

The Keetch-Byram Drought Index (Keetch & Byram, 1968) was developed for wildland fire-danger assessment and estimates the cumulative moisture deficit in the upper soil and duff layers from daily maximum temperature and precipitation, following the metric reformulation of Janis, Hubbard, and Redmond (2002).

$$DF = \{ [800 - KBDI_{(prev)}] \times [0.968 \times \exp(0.0875 \times Tmax + 1.5552) - 8.30] \times 10^{-3} \} / [1 + 10]$$

where Tmax is daily maximum temperature (°C), R is the region's long-term mean annual rainfall (cm), and KBDI_(prev) is the index value from the preceding day. Following a rainfall event, the accumulated wet-spell total is subtracted from the index (net of an initial abstraction of a few millimeters), reflecting the replenishment of soil moisture:

$$KBDI_t = KBDI_{(prev)} - 39.37 \times (wet\text{-}spell\text{ total, tenths-of-inch}) + DF$$

KBDI is bounded between 0 (fully saturated soil) and 800 (the maximum possible moisture deficit, expressed in hundredths of an inch), and is initialized following a rainfall event sufficient to bring the soil profile to saturation.

4. Hydrological Drought Indices

Hydrological drought indices characterize deficits in streamflow and surface water supply, and are therefore most directly relevant to water-resources management applications such as reservoir operation and irrigation allocation.

4.1 Streamflow Drought Index (SDI)

The Streamflow Drought Index (Nalbantis & Tsakiris, 2009) standardizes cumulative streamflow volumes within a hydrological year against the long-term distribution of that same cumulative period.

$$V_{i,k} = \sum_{j=1}^{3k} SF_{i,j}$$

i = hydrological year, j = month (1 = October), k = 1,2,3,4

$$SDI_{i,k} = (V_{i,k} - \bar{V}_k) / SD_k$$

where $SF_{i,j}$ is the streamflow volume in month j of hydrological year i , $V_{i,k}$ is the cumulative streamflow through the first $3k$ months of that hydrological year (giving four conventional reference periods — October-December, October-March, October-June, and the full October-September year), and $\bar{V}_k$ and SD_k are the long-term mean and standard deviation of V for reference period k .

4.2 Surface Water Supply Index (SWSI)

The Surface Water Supply Index was originally developed by Shafer and Dezman (1982) as a composite of snowpack, precipitation, streamflow, and reservoir storage, each standardized and combined with basin-specific weights; a widely used single-variable simplification based on streamflow alone, following Garen (1993), standardizes streamflow through its fitted cumulative probability:

$$SWSI = (100 \times P - 50) / 12$$

$$P = \text{GammaCDF}(\text{StreamFlow})$$

where *StreamFlow* is the observed streamflow for the month and location being evaluated, and *GammaCDF* denotes the cumulative distribution function of a gamma distribution fitted independently to that calendar month's streamflow across the full period of record.

5. Remote-Sensing Vegetation Drought Indices

Satellite-derived indices infer vegetation and soil-moisture stress from surface reflectance and land-surface temperature, offering spatially continuous drought monitoring independent of ground-based station networks.

5.1 Vegetation Health Index (VHI)

The Vegetation Health Index (Kogan, 1997) combines a vegetation-greenness signal with a thermal-stress signal into a single composite indicator.

$$VCI = 100 \times (NDVI - NDVI_{min}) / (NDVI_{max} - NDVI_{min})$$

$$TCI = 100 \times (LST_{max} - LST) / (LST_{max} - LST_{min})$$

$$VHI = w \times VCI + (1 - w) \times TCI$$

where *NDVI* is the normalized difference vegetation index and *LST* is land surface temperature for the period being evaluated, with minimum and maximum values taken across the full multi-year record for that calendar period; *VCI* and *TCI* are termed the Vegetation Condition Index and Temperature Condition Index respectively, and w is a weighting factor (conventionally 0.5) balancing their contributions.

5.2 Normalized Multiband Drought Index (NMDI)

The Normalized Multiband Drought Index (Wang & Qu, 2007) exploits the differential sensitivity of two shortwave-infrared bands to leaf and soil moisture content.

$$NMDI = [\rho_{NIR} - (\rho_{SWIR1} - \rho_{SWIR2})] / [\rho_{NIR} + (\rho_{SWIR1} - \rho_{SWIR2})]$$

where ρ_{NIR} , ρ_{SWIR1} ($\approx 1.64 \mu m$), and ρ_{SWIR2} ($\approx 2.13 \mu m$) are surface reflectance values in the near-infrared and two shortwave-infrared bands.

5.3 Visible and Shortwave-Infrared Drought Index (VSDI)

The Visible and Shortwave-Infrared Drought Index (Zhang, Chen, Zhu, & Zhou, 2013) combines visible-band and shortwave-infrared reflectance to jointly capture canopy water content and soil background moisture.

$$VSDI = 1 - [(\rho_{SWIR2} - \rho_{Blue}) + (\rho_{Red} - \rho_{Blue})]$$

where ρ_{Red} , ρ_{Blue} , and ρ_{SWIR2} are surface reflectance values in the red, blue, and shortwave-infrared-2 bands respectively.

6. Unit Conventions

Beyond the input conventions above, the following fixed conversions are applied wherever data must cross between the units in which it was supplied and the units a given calculation requires.

Temperature, Kelvin to Celsius:

$$T(^{\circ}C) = T(K) - 273.15$$

Temperature, Celsius to Fahrenheit (used internally by the Palmer-family and KBDI calculations, Sections 2.11 and 3.4):

$$T(^{\circ}F) = T(^{\circ}C) \times 9/5 + 32$$

Depth, millimeters to inches (used internally by the Palmer-family water balance):

$$d(\text{in}) = d(\text{mm}) / 25.4$$

Precipitation or evaporation flux to depth:

$$P(\text{mm/day}) = F(\text{kg}\cdot\text{m}^{-2}\cdot\text{s}^{-1}) \times 86,400$$

$$P(\text{mm/month}) = F(\text{kg}\cdot\text{m}^{-2}\cdot\text{s}^{-1}) \times 86,400 \times N \quad (N = \text{number of days} \in \text{the month})$$

Radiation to its equivalent daily magnitude:

$$X = R(\text{W}/\text{m}^2) \times 0.041674$$

General linear conversion, for cases not covered above:

$$X' = X \times k \text{ (scaling)} \quad X' = X - k \vee X' = X + k \text{ (offset)}$$

where k is a fixed constant appropriate to the specific quantity being converted.

7. Complementary Analytical Techniques

Beyond the indices themselves, several established statistical techniques are routinely applied to a computed drought-index time series to characterize its trend, periodicity, event structure, frequency behavior, and relationship to large-scale climate variability.

7.1 Trend Analysis

Trend detection identifies statistically significant long-term monotonic changes in a drought index over time. Four complementary methods are in widespread use.

Mann-Kendall test and Sen's slope

The Mann-Kendall test (Mann, 1945; Kendall, 1975) is a non-parametric test for monotonic trend that does not assume a particular data distribution, making it well suited to the often non-normal distributions of drought indices.

$$S = \sum_{i < j} \text{sign}(x_j - x_i)$$

$$\text{Var}(S) = [n(n-1)(2n+5) - \sum t(t-1)(2t+5)] / 18$$

$$Z = (S \mp 1) / \sqrt{\text{Var}(S)}, \quad p = 2 \times [1 - \Phi(|Z|)]$$

where t is the size of each group of tied values in the series. The magnitude of any detected trend is conventionally estimated by Sen's slope (Sen, 1968), the median of all pairwise slopes $(x_j - x_i)/(j - i)$ for $i < j$, a robust estimator insensitive to outliers.

Modified Mann-Kendall test

Because the standard Mann-Kendall test assumes independent observations, series with significant serial autocorrelation require a variance correction to avoid overstating trend significance (Hamed & Rao, 1998). The series is first detrended by Sen's slope, its autocorrelation structure at successive lags is estimated, and the Mann-Kendall variance is inflated accordingly before the significance test is repeated.

Spearman's rank correlation test

Spearman's rho provides an alternative non-parametric trend test based on the rank correlation between the observed values and their time order, with significance assessed via the Student's t-distribution on the transformed correlation coefficient.

Ordinary least-squares linear trend

A conventional linear regression of the index against time provides a parametric trend estimate (slope and intercept), with significance assessed via the F-distribution — informative when a linear trend model is an appropriate description of the series, but, unlike Mann-Kendall and Sen's slope, sensitive to outliers and non-linearity.

Innovative Trend Analysis (ITA)

Şen's (2012) Innovative Trend Analysis method splits the series into two equal halves, sorts each half independently, and compares them point-by-point, allowing trends to be visualized and tested across the low, medium, and high portions of a distribution rather than as a single aggregate slope — a distinction the preceding methods do not offer.

$$D = (\text{mean}_{2nd_half} - \text{mean}_{1st_half}) / (SD_{1st_half} / \sqrt{m})$$

with a trend considered significant when $|D|$ exceeds 1.96, the standard 95%-confidence threshold of the normal distribution.

7.2 Wavelet Spectral Analysis

Wavelet analysis decomposes a time series simultaneously in time and frequency (or, equivalently, time and period), revealing whether and when particular periodicities — such as multi-year drought cycles associated with ENSO — are present, and whether their strength changes over the record. The continuous wavelet transform using the Morlet mother wavelet (Torrence & Compo, 1998) is the standard approach in climate applications.

$$\psi_0(\eta) = \pi^{1/4} \times e^{i\omega_0 \eta} \times e^{-\eta^2/2}$$

$$period = 4\pi \times scale / (\omega_0 + \sqrt{2 + \omega_0^2})$$

where ω_0 (conventionally 6) sets the wavelet's characteristic oscillation, and scale is varied to sweep across periods of interest. The time-averaged wavelet power across the full record — the global wavelet spectrum — identifies which periodicities dominate overall, and statistical significance is assessed against a red-noise (lag-1 autoregressive) background spectrum, following Torrence and Compo's (1998) formulation, so that spurious peaks arising from persistence alone can be distinguished from genuine periodic behavior.

7.3 Drought Event and Severity Analysis

Run theory (Yevjevich, 1967) formalizes the extraction of discrete drought events from a continuous index series by defining an event as any maximal run of consecutive periods in which the index remains below (or above) a chosen threshold.

$$Duration = \text{number of periods} \in \text{the run}$$

$$Severity = \sum |index - threshold| \text{ -the run}$$

$$Peak \text{ intensity} = \text{most extreme index value within the run}$$

Return periods for the ranked severity of historical events are commonly estimated from the Weibull plotting-position formula, $T = (N+1)/m$, where N is the total number of events on record and m is an event's rank when sorted by descending severity.

7.4 Severity-Duration-Frequency (SDF) Analysis

Because drought duration and severity are correlated rather than independent, a bivariate frequency analysis provides a more realistic joint return period than treating either variable alone (Shiau, 2006). Duration and severity, extracted from historical drought events, are each fitted to a gamma distribution, and their dependence structure is captured by a Gumbel-Hougaard copula, whose single parameter is derived from Kendall's rank correlation, τ , between the two variables:

$$\theta = 1 / (1 - \tau)$$

$$C(u, v) = \exp \{ -[(-\ln u)^\theta + (-\ln v)^\theta]^{1/\theta} \}$$

where u and v are the marginal cumulative probabilities of duration and severity respectively. From the joint distribution, both a joint-AND return period (duration and severity both equalled or exceeded) and a joint-OR return period (either equalled or exceeded) can be derived for any combination of duration and severity, from which severity-duration-frequency curves are constructed.

7.5 Teleconnection Analysis

Large-scale climate oscillations — the El Niño-Southern Oscillation (ENSO), the Pacific Decadal Oscillation (PDO), and the North Atlantic Oscillation (NAO) among them — are well documented to modulate regional drought occurrence (Ropelewski & Halpert, 1987; Mantua & Hare, 2002). Their influence is conventionally assessed through lagged cross-correlation between a drought index and a chosen teleconnection index, evaluated across a range of lead/lag months to identify both the strength and the characteristic lag of the relationship:

$$r(L) = \Sigma(dx \cdot dy) / \sqrt{(\Sigma dx^2 \cdot \Sigma dy^2)}$$

where L is the lag in months and dx, dy are the deviations of the two series from their respective means over the paired sample; both Pearson and Spearman forms of the correlation are typically examined, with significance assessed via the Student's t-distribution, and the lag of largest absolute correlation identified as the dominant lead/lag relationship between the two series.

8. Drought Classification Scales

Each index is conventionally interpreted against a classification scale established in its originating literature, reproduced below for reference.

8.1 SPI and related standardized indices (CZI, MCZI, ZSI, SDI, standardized RDI)

McKee, Doesken, and Kleist's (1993) classification for SPI is, by long-standing convention, applied identically to any other index expressed on a comparable standardized (approximately zero-mean, unit-variance) scale.

Value range	Classification
≥ 2.00	Extremely wet
1.50 to 1.99	Severely wet
1.00 to 1.49	Moderately wet
-0.99 to 0.99	Near normal
-1.49 to -1.00	Moderately dry
-1.99 to -1.50	Severely dry
≤ -2.00	Extremely dry

8.2 Effective Drought Index (EDI)

Value range	Classification
-1 to 1	Near-normal conditions
< -2	Severe drought

8.3 Palmer family (PDSI, PHDI, Self-Calibrated PDSI)

Palmer's (1965) original classification remains the most widely reproduced scale in operational drought climatology.

Value range	Classification
≥ 4.00	Extremely wet
3.00 to 3.99	Very wet
2.00 to 2.99	Moderately wet
1.00 to 1.99	Slightly wet
0.50 to 0.99	Incipient wet spell
-0.49 to 0.49	Near normal
-0.99 to -0.50	Incipient dry spell
-1.99 to -1.00	Mild drought
-2.99 to -2.00	Moderate drought
-3.99 to -3.00	Severe drought
≤ -4.00	Extreme drought

8.4 Soil Moisture Deficit Index and Evapotranspiration Deficit Index

Both indices are deliberately scaled (Narasimhan & Srinivasan, 2005) to range approximately -4 to +4, for direct visual comparison with the Palmer classification in Section 8.3.

8.5 Keetch-Byram Drought Index (KBDI)

Value range (0.01 in)	mm equivalent	Interpretation
0 – 200	0 – 50	Soil moist; 0 = completely saturated
200 – 400	50 – 100	Leaf litter begins to dry
400 – 600	100 – 150	Lower litter actively contributes to fire intensity
600 – 800	150 – 200	Severe drought, extreme fire behavior

8.6 Percent of Normal Precipitation (PN)

Willeke, Hosking, Wallis, and Guttman (1994) define regional and seasonal PN thresholds in the National Drought Atlas rather than a single universal cutoff; values below 100 percent indicate below-normal precipitation, and values below approximately 80 percent are conventionally read as drought, subdivided into four increasing-severity classes whose precise cutoffs vary by region and season.

8.7 Agricultural Reconnaissance Index (ARI)

Value range	Classification
< 40	Drought conditions

Value range	Classification
40 to 200	Favorable for vegetation growth and agricultural productivity
> 200	Elevated moisture / wet conditions

8.8 Deciles Index (DI)

Deciles	Classification
1 – 2 (driest 20%)	Much below normal
3 – 4	Below normal
5 – 6	Near normal
7 – 8	Above normal
9 – 10 (wettest 20%)	Much above normal

8.9 Vegetation Health Index (VHI)

On VHI's native 0–100 scale (Kogan, 1997):

Value range	Classification
≤ 10	Extreme vegetation stress / drought
10 – 20	Severe stress
20 – 30	Moderate stress
30 – 40	Mild stress
> 40	No stress (favorable conditions)

9. Concluding Remarks

No single drought index is universally superior; each embeds assumptions — about the underlying probability distribution, the relevant accumulation period, the role of temperature and evapotranspiration, or the soil-moisture processes being represented — that make it better suited to some applications than others. Meteorological indices such as SPI remain the most broadly applicable owing to their minimal data requirements, but increasingly are used alongside evapotranspiration-sensitive indices such as SPEI and RDI to capture the influence of rising temperatures on drought severity, a relationship precipitation-only indices cannot represent. Agricultural and hydrological indices provide sector-specific detail once soil-moisture or streamflow observations are available, while satellite-derived vegetation indices extend monitoring to regions lacking dense ground-station networks. The complementary techniques of Section 7 — trend detection, spectral analysis, event and frequency characterization, and teleconnection analysis — allow any of these indices to be examined not only as a snapshot of current conditions but as part of a longer-term climatological and large-scale-circulation context, which is typically the more actionable frame for drought risk management and planning.

10. References

- Abramowitz, M., & Stegun, I. A. (1965). Handbook of mathematical functions with formulas, graphs, and mathematical tables. New York: Dover Publications.
- Alley, W. M. (1984). The Palmer Drought Severity Index: Limitations and assumptions. *Journal of Climate and Applied Meteorology*, 23(7), 1100–1109.
- Byun, H. R., & Wilhite, D. A. (1999). Objective quantification of drought severity and duration. *Journal of Climate*, 12(9), 2747–2756.
- Edwards, D. C., & McKee, T. B. (1997). Characteristics of 20th century drought in the United States at multiple time scales (Climatology Report 97-2). Fort Collins, CO: Colorado State University.
- Garen, D. C. (1993). Revised surface-water supply index for western United States. *Journal of Water Resources Planning and Management*, 119(4), 437–454.
- Gibbs, W. J., & Maher, J. V. (1967). Rainfall deciles as drought indicators (Bulletin No. 48). Melbourne: Australian Bureau of Meteorology.
- Hamed, K. H., & Rao, A. R. (1998). A modified Mann-Kendall trend test for autocorrelated data. *Journal of Hydrology*, 204(1–4), 182–196.
- Hosking, J. R. M. (1990). L-moments: Analysis and estimation of distributions using linear combinations of order statistics. *Journal of the Royal Statistical Society, Series B*, 52(1), 105–124.
- Jacobi, J. M., Perrone, D., Duncan, L. L., & Hornberger, G. (2013). A tool for calculating the Palmer drought indices. *Water Resources Research*, 49(9), 6086–6089.
- Janis, M. J., Hubbard, K. G., & Redmond, K. T. (2002). Determination of the Keetch-Byram Drought Index air temperature for monitoring the Keetch-Byram Drought Index. In *Proceedings of the 13th Conference on Applied Climatology*. Portland, OR: American Meteorological Society.
- Keetch, J. J., & Byram, G. M. (1968). A drought index for forest fire control (Research Paper SE-38). Asheville, NC: USDA Forest Service, Southeastern Forest Experiment Station.
- Kendall, M. G. (1975). Rank correlation methods (4th ed.). London: Charles Griffin.
- Kogan, F. N. (1997). Global drought watch from space. *Bulletin of the American Meteorological Society*, 78(4), 621–636.
- Mann, H. B. (1945). Nonparametric tests against trend. *Econometrica*, 13(3), 245–259.
- Mantua, N. J., & Hare, S. R. (2002). The Pacific Decadal Oscillation. *Journal of Oceanography*, 58(1), 35–44.
- McKee, T. B., Doesken, N. J., & Kleist, J. (1993). The relationship of drought frequency and duration to time scales. In *Proceedings of the 8th Conference on Applied Climatology* (pp. 179–184). Anaheim, CA: American Meteorological Society.
- McKee, T. B., Doesken, N. J., & Kleist, J. (1995). Drought monitoring with multiple time scales. In *Proceedings of the 9th Conference on Applied Climatology* (pp. 233–236). Dallas, TX: American Meteorological Society.
- Mishra, A. K., & Singh, V. P. (2010). A review of drought concepts. *Journal of Hydrology*, 391(1–2), 202–216.
- Nalbantis, I., & Tsakiris, G. (2009). Assessment of hydrological drought revisited. *Water Resources Management*, 23(5), 881–897.
- Narasimhan, B., & Srinivasan, R. (2005). Development and evaluation of Soil Moisture Deficit Index (SMDI) and Evapotranspiration Deficit Index (ETDI) for agricultural drought monitoring. *Agricultural and Forest Meteorology*, 133(1–4), 69–88.
- Nieuwolt, S. (1981). Agricultural droughts in the tropics. In J. Gribbin (Ed.), *Climatic variations and variability: Facts and theories* (pp. 449–464). Dordrecht: D. Reidel Publishing.

- Palmer, W. C. (1965). Meteorological drought (Research Paper No. 45). Washington, DC: U.S. Weather Bureau.
- Ropelewski, C. F., & Halpert, M. S. (1987). Global and regional scale precipitation patterns associated with the El Niño/Southern Oscillation. *Monthly Weather Review*, 115(8), 1606–1626.
- Sen, P. K. (1968). Estimates of the regression coefficient based on Kendall's tau. *Journal of the American Statistical Association*, 63(324), 1379–1389.
- Şen, Z. (2012). Innovative trend analysis methodology. *Journal of Hydrologic Engineering*, 17(9), 1042–1046.
- Shafer, B. A., & Dezman, L. E. (1982). Development of a Surface Water Supply Index (SWSI) to assess the severity of drought conditions in snowpack runoff areas. In *Proceedings of the Western Snow Conference* (pp. 164–175). Reno, NV.
- Shiau, J. T. (2006). Fitting drought duration and severity with two-dimensional copulas. *Water Resources Management*, 20(5), 795–815.
- Thom, H. C. S. (1958). A note on the gamma distribution. *Monthly Weather Review*, 86(4), 117–122.
- Torrence, C., & Compo, G. P. (1998). A practical guide to wavelet analysis. *Bulletin of the American Meteorological Society*, 79(1), 61–78.
- Tsakiris, G., & Vangelis, H. (2005). Establishing a drought index incorporating evapotranspiration. *European Water*, 9(10), 3–11.
- Van Rooy, M. P. (1965). A rainfall anomaly index independent of time and space. *Notos*, 14, 43–48.
- Vicente-Serrano, S. M., Beguería, S., & López-Moreno, J. I. (2010). A multiscalar drought index sensitive to global warming: The Standardized Precipitation Evapotranspiration Index. *Journal of Climate*, 23(7), 1696–1718.
- Wang, L., & Qu, J. J. (2007). NMDI: A normalized multi-band drought index for monitoring soil and vegetation moisture with satellite remote sensing. *Geophysical Research Letters*, 34(20), L20405.
- Wells, N., Goddard, S., & Hayes, M. J. (2004). A self-calibrating Palmer Drought Severity Index. *Journal of Climate*, 17(12), 2335–2351.
- Wilhite, D. A., & Glantz, M. H. (1985). Understanding the drought phenomenon: The role of definitions. *Water International*, 10(3), 111–120.
- Willeke, G., Hosking, J. R. M., Wallis, J. R., & Guttman, N. B. (1994). *The National Drought Atlas* (Institute for Water Resources Report 94-NDS-4). Alexandria, VA: U.S. Army Corps of Engineers.
- World Meteorological Organization. (2012). *Standardized Precipitation Index user guide* (WMO-No. 1090). Geneva: WMO.
- Wu, H., Hayes, M. J., Weiss, A., & Hu, Q. (2001). An evaluation of the Standardized Precipitation Index, the China-Z Index and the statistical Z-Score. *International Journal of Climatology*, 21(6), 745–758.
- Yevjevich, V. (1967). *An objective approach to definitions and investigations of continental hydrologic droughts* (Hydrology Papers No. 23). Fort Collins, CO: Colorado State University.
- Zhang, N., Chen, S., Zhu, W., & Zhou, Q. (2013). VSDI: A visible and shortwave infrared drought index for monitoring soil and vegetation moisture. *International Journal of Remote Sensing*, 34(13), 4585–4609.