

# AgriMetSoft

## Formulas in SD-GCM V2.1

Statistical Downscaling & Bias Correction of GCM Data

<https://agrimetsoft.com/sd-gcm>

### What is new in V2.1

Three new bias-correction methods: QDM, DQM, SDM.

Monthly Stratification is an optional checkbox available for all six methods.

LOCI (Local Intensity Scaling) pre-processing applied automatically to all precipitation methods.

Wavelet Downscaling has been removed.

Six new evaluation metrics: KGE, PBIAS, RSR, Wet-Day Frequency Ratio, Spearman  $\rho$ , NRMSE.

Teal-shaded cells indicate content that is new or changed in V2.1.

# 1. Introduction

SD-GCM V2.1 provides six methods for statistically downscaling and bias-correcting Global Climate Model (GCM) output to the scale of local station observations. This document defines the mathematical formulas for all methods.

Every method works in two correction modes, chosen by the user in the tool:

- Additive mode: for temperature variables (daily mean, minimum, or maximum temperature). The correction is added to or subtracted from the GCM value.
- Multiplicative mode: for precipitation and other ratio-scale variables (rainfall, evapotranspiration, solar radiation). The correction is a scaling factor. Negative outputs are always set to zero.

#	Abbrev.	Full Name	Reference
0	Delta	Delta Method (Scaling)	Panofsky & Brier (1968)
1	QM	Quantile Mapping	Gudmundsson et al. (2012); Wood et al. (2002)
2	EQM	Empirical Quantile Mapping	Boé et al. (2007); Wetterhall et al. (2012)
3	QDM	Quantile Delta Mapping	Cannon, Sobie & Murdock (2015)
4	DQM	Detrended Quantile Mapping	Cannon, Sobie & Murdock (2015)
5	SDM	Scaled Distribution Mapping	Switanek et al. (2017)

## 2. Variable Names and Symbols

All formulas use the following variable names. These names appear in the equations throughout this document.

Symbol	Meaning for the User
$V_{Obs}(t)$	Your observed station data value at time step $t$ (e.g. daily rainfall or temperature at the weather station)
$V_{GCM-Hist}(t)$	Historical GCM output value at time step $t$ (the GCM simulation of the past climate, same period as your observations)
$V_{GCM-SSP}(t)$	Future GCM output value at time step $t$ (the GCM projection under the selected SSP or RCP scenario)
$V_{Corrected}(t)$	Bias-corrected output value at time step $t$ (the final result written to your output file)
$\mu_{Obs}$	Long-term mean of your observed station data over the calibration period
$\mu_{GCM-Hist}$	Long-term mean of historical GCM data over the calibration period
$\mu_{GCM-SSP}$	Long-term mean of future GCM data over the projection period
$\mu_{Obs,m}$	Monthly mean of observed data for calendar month $m$ (e.g. mean of all January values)
$\mu_{GCM-Hist,m}$	Monthly mean of historical GCM data for calendar month $m$
$\sigma_{Obs}$	Standard deviation of observed station data
$\sigma_{GCM-Hist}$	Standard deviation of historical GCM data
$\delta$	Correction factor (delta): a single value computed from the full calibration period (Global mode)
$\delta_m$	Monthly correction factor: one value per calendar month $m$ (Monthly Stratification mode)
$CDF_{Obs}(v)$	Cumulative distribution function of observed station data: the fraction of observed values that are $\leq v$
$CDF_{GCM-Hist}(v)$	Cumulative distribution function of historical GCM data: the fraction of historical GCM values that are $\leq v$
$CDF^{-1}_{Obs}(p)$	Inverse CDF of observed data: the station value that corresponds to cumulative probability $p$
$p$	Cumulative probability, a number between 0 and 1
$p_{exceed}$	Exceedance probability = $1 - p$ (probability of exceeding a given value)
$\Delta(t)$	Quantile delta: the GCM-projected change at quantile $p(t)$ , either additive (temperature) or multiplicative (precipitation)

$\theta_{wet}$	Wet-day threshold = 0.1 mm/day. Days with rainfall below this are counted as dry days.
$WetFreq_{Obs}$	Fraction of wet days in observed data = (number of observed days with rainfall > $\theta_{wet}$ ) / (total observed days)
$WetFreq_{GCM}$	Fraction of wet days in GCM data before LOCI correction
$Threshold_{LOCI}$	The GCM rainfall threshold found by LOCI so that the GCM wet-day frequency matches $WetFreq_{Obs}$
$ScaleFactor_{LOCI}$	Intensity scaling factor in LOCI = (mean observed wet-day rainfall) / (mean GCM wet-day rainfall above $Threshold_{LOCI}$ )
$V_{LOCI}(t)$	GCM value after LOCI pre-processing: dry days set to 0, wet days scaled by $ScaleFactor_{LOCI}$
$m$	Calendar month number: 1 = January, 2 = February, ..., 12 = December
$t$	Time step index (day, month, or 3-hour interval depending on your input data)
$n$	Number of values in a sorted data array used for CDF construction

### 3. Unit Conversion

Before any bias-correction method runs, the raw GCM values from the NetCDF file are converted to the same physical units as your observed station data. You select the conversion in the Gridded Data tab. The tool applies the same conversion to both the historical GCM data and the future SSP/RCP data.

Code	Variable Type	Formula	Typical Use
—	No change	$V' = V$	Data already in the correct units
K→°C	Temperature	$V' = V - 273.15$	Converts Kelvin to Celsius (tas, tasma, tasmin)
kg/m <sup>2</sup> s→mm/d	Precipitation	$V' = V \times 86\,400$	Converts flux to mm per day (pr, prsn)
W/m <sup>2</sup> →h/d	Solar radiation	$V' = V \times 0.041\,674$	Converts W/m <sup>2</sup> to sunshine hours per day (rsds)
× b	Custom multiply	$V' = V \times b$	Example: Pa → hPa (×0.01)
− b	Custom subtract	$V' = V - b$	Custom offset removal
+ b	Custom add	$V' = V + b$	Custom offset addition

## 4. A — Delta Method

Reference: Panofsky & Brier (1968).

The Delta method corrects the GCM output by multiplying (for precipitation) or shifting (for temperature) each future GCM value by a correction factor  $\delta$ . This factor is the ratio or difference between the long-term mean of the observed station data and the long-term mean of the historical GCM data.

### Global mode vs Monthly Stratification mode

Global mode (Monthly Stratification checkbox OFF): one correction factor  $\delta$  is computed using all months of the calibration period together.

Monthly Stratification mode (checkbox ON): twelve separate correction factors  $\delta_m$  are computed, one for each calendar month. For example,  $\delta_{\text{Jan}}$  uses only January data from both observed and historical GCM.

This choice applies to all six methods — see Section 12.

### 4.1 Step 1 — Compute the Correction Factor

Global mode (one  $\delta$  for the whole year):

$\delta = \mu_{\text{Obs}} / \mu_{\text{GCM-Hist}}$ (multiplicative, for precipitation)	<b>(1a)</b>
$\delta = \mu_{\text{Obs}} - \mu_{\text{GCM-Hist}}$ (additive, for temperature)	<b>(1b)</b>

Monthly Stratification mode (one  $\delta_m$  per calendar month):

$\delta_m = \mu_{\text{Obs},m} / \mu_{\text{GCM-Hist},m}$ (multiplicative)	<b>(2a)</b>
$\delta_m = \mu_{\text{Obs},m} - \mu_{\text{GCM-Hist},m}$ (additive)	<b>(2b)</b>

### 4.2 Step 2 — Apply the Correction to the Future GCM Data (Bias Correction tab)

$V_{\text{Corrected}}(t) = \max(0, V_{\text{GCM-SSP}}(t) \times \delta_m(t))$ (multiplicative)	<b>(3a)</b>
$V_{\text{Corrected}}(t) = V_{\text{GCM-SSP}}(t) + \delta_m(t)$ (additive)	<b>(3b)</b>

### 4.3 Apply the Correction to the Historical GCM Data (Evaluation tab)

$V_{\text{Corrected}}(t) = \max(0, V_{\text{GCM-Hist}}(t) \times \delta_m(t))$ (multiplicative)	<b>(4a)</b>
$V_{\text{Corrected}}(t) = V_{\text{GCM-Hist}}(t) + \delta_m(t)$ (additive)	<b>(4b)</b>

In both cases,  $\delta_m(t)$  means the correction factor for the calendar month of time step  $t$ . In Global mode,  $\delta_m(t) = \delta$  (the same value for all months).

## 5. B — Quantile Mapping (QM)

References: Gudmundsson et al. (2012); Wood et al. (2002).

Quantile Mapping corrects the GCM data so that its statistical distribution matches the distribution of the observed station data. Instead of matching only the mean (as Delta does), QM matches the full range of values — low values, typical values, and high values alike.

The idea: for each GCM value, find what fraction of historical GCM values it exceeds (its cumulative probability  $p$ ). Then find the observed station value that the same fraction  $p$  of observations falls below. Use that observed value as the bias-corrected output.

### Key difference between QM and EQM in V2.1

Temperature: QM uses a Normal (Gaussian) probability distribution to describe both the observed and historical GCM data. EQM uses the actual data directly without assuming any distribution.

Precipitation: Both QM and EQM use the same empirical (data-driven) approach in V2.1. For precipitation, the two methods are mathematically identical.

Practical guidance: for temperature, EQM is more flexible and does not require the data to follow a bell curve. QM for temperature is suitable when the data is approximately Normal.

### 5.1 Temperature — QM with Normal Distribution

QM fits a Normal distribution to both the observed station data and the historical GCM data. The correction maps each GCM value through the historical Normal CDF, then through the inverse of the observed Normal CDF:

<i>Step 1: Find the cumulative probability of the GCM value:</i>	(—)
$p(t) = \Phi_t(V\_GCM-Hist(t); \mu\_GCM-Hist, \sigma\_GCM-Hist)$	(5a)
<i>Step 2: Find the corrected value from the observed distribution:</i>	(—)
$V\_Corrected(t) = \Phi^{-1}\_Obs(p(t); \mu\_Obs, \sigma\_Obs)$	(5b)

where  $\Phi_t(v; \mu, \sigma)$  is the Normal cumulative distribution function with mean  $\mu$  and standard deviation  $\sigma$ , and  $\Phi^{-1}\_Obs$  is its inverse (the quantile function of the observed Normal distribution).

### 5.2 Precipitation — QM with Empirical Distribution (V2.1)

For precipitation, QM uses the actual data values sorted in order, without fitting a mathematical distribution. LOCI pre-processing is applied first to correct the GCM's wet-day frequency (see Section 10):

<i>Step 1: Pre-process the GCM data with LOCI (Section 10):</i>	(—)
$V\_LOCI(t) = LOCI(V\_GCM-Hist(t))$	(6a)
<i>Step 2: Find what fraction of historical GCM wet days are <math>\leq V\_LOCI(t)</math>:</i>	(—)

$$p(t) = CDF_{GCM-Hist}(V_{LOCI}(t))$$

(6b)

Step 3: Find the corrected value from the observed data at probability  $p$ :

(—)

$$V_{Corrected}(t) = CDF^{-1}_{Obs}(p(t))$$

(6c)

$$V_{Corrected}(t) = 0 \text{ if } V_{LOCI}(t) \leq \theta_{wet} \text{ (dry day)}$$

(6d)

**Change from V2.0:** In V2.0, precipitation QM used a Gamma probability distribution. In V2.1 it uses the actual sorted data values (empirical approach), which is more robust across different climates.

## 6. C — Empirical Quantile Mapping (EQM)

References: Boé et al. (2007); Wetterhall et al. (2012).

EQM works in the same way as QM but uses the actual data values (sorted in order) for both temperature and precipitation — no probability distribution is assumed. This makes EQM fully non-parametric: it works directly with what the data shows.

### 6.1 Temperature — EQM with Empirical Distribution

For each historical GCM temperature value, find its rank among all historical GCM values to determine  $p$ , then find the observed temperature at the same rank:

<i>Step 1: Count how many historical GCM values are <math>\leq V\_GCM-Hist(t)</math>:</i>	<b>(—)</b>
$p(t) = CDF\_GCM-Hist(V\_GCM-Hist(t))$ [using sorted data]	<b>(7a)</b>
<i>Step 2: Find the observed station value at the same probability:</i>	<b>(—)</b>
$V\_Corrected(t) = CDF^{-1}\_Obs(p(t))$ [using sorted data]	<b>(7b)</b>

### 6.2 Precipitation — EQM

For precipitation, the algorithm is the same as QM precipitation (Section 5.2) with LOCI pre-processing. The formulas are identical.

### 6.3 Summary: QM vs EQM

Aspect	QM vs EQM comparison
Temperature	QM uses a Normal (bell curve) distribution fitted to the data. EQM uses the actual sorted data values directly. EQM makes no assumptions about the shape of the distribution.
Precipitation	Identical formulas and results in V2.1. Both use LOCI pre-processing followed by empirical (data-driven) quantile mapping.
When to choose QM	When your temperature data is approximately bell-shaped (Normally distributed). Slightly smoother results for small datasets.
When to choose EQM	When your temperature data may not be Normally distributed (skewed, multi-modal). Always equivalent to QM for precipitation.

## 7. D — Quantile Delta Mapping (QDM)

Reference: Cannon, Sobie & Murdock (2015) *J. Climate* 28:6938–6959. **[NEW IN V2.1]**

QDM is designed for future climate projections. It not only corrects the bias in the GCM data (like QM and EQM do) but also preserves the climate change signal that the GCM is projecting for every part of the distribution — not just the average change.

The key idea: for each future GCM value, compute how much it differs from the historical GCM at the same rank (the “quantile delta”). Add that projected change on top of the observed station value at the same rank. This way the correction uses observed climatology but retains the full GCM change signal.

### 7.1 Temperature — QDM (Additive)

<i>Step 1: Find the rank (probability) of the future GCM value among all future GCM values:</i>	(—)
$p(t) = CDF_{GCM-SSP}(V_{GCM-SSP}(t))$	(8a)
<i>Step 2: Find the historical GCM value at the same probability p:</i>	(—)
$V_{GCM-Hist\ at\ p} = CDF^{-1}_{GCM-Hist}(p(t))$	(8b)
<i>Step 3: Compute the projected temperature change (quantile delta):</i>	(—)
$\Delta(t) = V_{GCM-SSP}(t) - V_{GCM-Hist\ at\ p} \text{ [warming or cooling signal]}$	(8c)
<i>Step 4: Add the projected change to the observed station value at probability p:</i>	(—)
$V_{Corrected}(t) = CDF^{-1}_{Obs}(p(t)) + \Delta(t)$	(8d)

### 7.2 Precipitation — QDM (Multiplicative)

LOCI pre-processing is applied to both historical and future GCM data before the quantile mapping:

<i>Step 1: Pre-process both GCM datasets with LOCI (Section 10)</i>	(—)
<i>Step 2: <math>p(t) = CDF_{GCM-SSP-LOCI}(V_{LOCI-SSP}(t))</math></i>	(9a)
<i>Step 3: <math>V_{GCM-Hist\ at\ p} = CDF^{-1}_{GCM-Hist-LOCI}(p(t))</math></i>	(9b)
<i>Step 4: <math>\Delta(t) = V_{LOCI-SSP}(t) / V_{GCM-Hist\ at\ p}</math> [rainfall intensification ratio]</i>	(9c)
<i>Step 5: <math>V_{Corrected}(t) = \max(0, CDF^{-1}_{Obs}(p(t)) \times \Delta(t))</math></i>	(9d)
$V_{Corrected}(t) = 0 \text{ if } V_{LOCI-SSP}(t) \leq \theta_{wet} \text{ (dry day)}$	(9e)

If  $V_{GCM-Hist\ at\ p}$  is near zero ( $< 10^{-10}$ ), the ratio  $\Delta(t)$  is set to 1 to avoid numerical instability.

## 8. E — Detrended Quantile Mapping (DQM)

Reference: Cannon, Sobie & Murdock (2015) *J. Climate* 28:6938–6959. **[NEW IN V2.1]**

DQM is a simpler alternative to QDM. Rather than preserving the change signal at every quantile, it preserves only the average (mean) change. The approach: temporarily remove the average trend from the future GCM data, apply EQM to the detrended data (so the bias correction sees a distribution similar to the historical period), then add the trend back.

### 8.1 Temperature — DQM (Additive)

<i>Step 1: Compute the average warming trend:</i>	(—)
$Trend = \mu_{GCM-SSP} - \mu_{GCM-Hist}$	<b>(10a)</b>
<i>Step 2: Remove the trend from each future GCM value:</i>	(—)
$V_{Detrended}(t) = V_{GCM-SSP}(t) - Trend$	<b>(10b)</b>
<i>Step 3: Apply EQM temperature correction (Section 6.1) to the detrended data:</i>	(—)
$V_{EQM}(t) = EQM(V_{Detrended}(t))$	<b>(10c)</b>
<i>Step 4: Add the trend back to restore the projected warming:</i>	(—)
$V_{Corrected}(t) = V_{EQM}(t) + Trend$	<b>(10d)</b>

### 8.2 Precipitation — DQM (Multiplicative)

The trend is expressed as the ratio of wet-day means. LOCI pre-processing is applied inside the EQM step:

<i>Step 1: Compute the average intensification ratio:</i>	(—)
$Ratio = \mu_{GCM-Hist,wet} / \mu_{GCM-SSP,wet}$ [ratio < 1 means future is wetter]	<b>(11a)</b>
<i>Step 2: Scale down future wet-day values to remove the trend:</i>	(—)
$V_{Detrended}(t) = V_{GCM-SSP}(t) \times Ratio$ [if $V_{GCM-SSP}(t) > \theta_{wet}$ ]	<b>(11b)</b>
<i>Step 3: Apply EQM precipitation correction (Section 6.2) to the detrended data</i>	<b>(11c)</b>
<i>Step 4: Reapply the trend to restore projected intensification:</i>	(—)
$V_{Corrected}(t) = \max(0, V_{EQM}(t) / Ratio)$	<b>(11d)</b>

#### DQM vs QDM: which should you use?

DQM: preserves only the mean change. Simpler and faster. Good when you mainly care about changes in average precipitation or temperature.

QDM: preserves the change at every quantile of the distribution. Better when you need to study changes in extremes (heavy rainfall events, heat waves, droughts).  
Both are better than EQM alone for future projections because EQM can suppress the climate change signal embedded in the GCM.

## 9. F — Scaled Distribution Mapping (SDM)

Reference: Switanek et al. (2017) *Hydrol. Earth Syst. Sci.* 21:2649–2666. [\[NEW IN V2.1\]](#)

SDM corrects both the distribution shape and the frequency of wet days simultaneously. It maps future GCM values using exceedance probability (the probability of exceeding a value) rather than cumulative probability. For precipitation, it explicitly solves the “drizzle bias” problem: GCMs often produce too many light-rain days that don’t occur in reality.

### 9.1 Temperature — SDM

<i>Step 1: Find the exceedance probability of the future GCM value in the historical GCM distribution:</i>	(—)
$p_{\text{exceed}}(t) = 1 - \text{CDF}_{\text{GCM-Hist}}(V_{\text{GCM-SSP}}(t))$	(12a)
<i>Step 2: Map to the observed distribution using the same exceedance probability:</i>	(—)
$V_{\text{Corrected}}(t) = \text{CDF}^{-1}_{\text{Obs}}(1 - p_{\text{exceed}}(t))$	(12b)

### 9.2 Precipitation — SDM (Wet-Day Frequency Correction)

SDM ranks all future GCM wet days by intensity (lightest to heaviest). If the GCM produces more wet days than observed, the lightest-intensity excess days are set to zero. The remaining wet days are mapped to the observed distribution:

<i>Step 1: Rank all future GCM wet days from lightest to heaviest intensity</i>	(13a)
<i>Step 2a: <math>n_{\text{excess}} = \max(0, n_{\text{GCM-wet}} - n_{\text{Obs-wet}})</math></i>	(13b)
<i>Step 2b: Set the lightest <math>n_{\text{excess}}</math> wet days to 0 (reclassify as dry days)</i>	(13c)
<i>Step 3: Map each remaining wet day to the observed distribution by rank:</i>	(—)
$\text{Rank\_in\_subset}(k) = k - n_{\text{excess}} + 1 \quad (1\text{-based})$	(13d)
$\text{Observed index} = \text{round}[(\text{Rank\_in\_subset} - 1) / (n_{\text{mapped}} - 1) \times (n_{\text{Obs-wet}} - 1)]$	(13e)
$V_{\text{Corrected}}(k) = \max(0, V_{\text{Obs,sorted}}[\text{Observed index }])$	(13f)

where  $n_{\text{GCM-wet}}$  = number of future GCM wet days,  $n_{\text{Obs-wet}}$  = number of observed wet days,  $n_{\text{mapped}} = \min(n_{\text{GCM-wet}}, n_{\text{Obs-wet}})$ .

#### How SDM handles the drizzle bias

If the GCM has MORE wet days than observations: the lightest  $n_{\text{excess}}$  wet days become dry. Only the remaining (heavier) days are bias-corrected.

If the GCM has FEWER wet days than observations: all GCM wet days are mapped; the rank scaling stretches them across the full observed range.

Intensity order is always preserved: the lightest GCM wet days map to the lightest observed wet days.

## 10. LOCI — Wet-Day Frequency Pre-Processor

Reference: Schmidli, Frei & Vidale (2006) *Int. J. Climatol.* 26:679–689. [\[NEW IN V2.1\]](#)

LOCI stands for Local Intensity Scaling. It is automatically applied to GCM precipitation data before any quantile-mapping method (QM, EQM, QDM, DQM). You do not need to activate it separately. LOCI is not applied to temperature.

The problem LOCI solves: GCMs typically produce too many light-rain days (the “drizzle bias”). Before fitting the transfer function, LOCI adjusts the GCM data so that its wet-day frequency and mean wet-day intensity match the observed station data.

<i>Step 1: Find the GCM rainfall threshold that gives the correct wet-day frequency:</i>	(—)
<i>Threshold_LOCI = the GCM value at rank (1 – WetFreq_Obs) of all GCM values</i>	<b>(14a)</b>
<i>Threshold_LOCI = max(Threshold_LOCI, <math>\theta_{wet}</math>)</i>	<b>(14b)</b>
<i>Step 2: Compute mean GCM rainfall above the threshold:</i>	(—)
<i><math>\mu_{GCM-wet,above}</math> = mean of all GCM values &gt; Threshold_LOCI</i>	<b>(14c)</b>
<i>Step 3: Compute the intensity scaling factor:</i>	(—)
<i>ScaleFactor_LOCI = (mean observed wet-day rainfall) / <math>\mu_{GCM-wet,above}</math></i>	<b>(14d)</b>
<i>Step 4: Apply to every GCM value:</i>	(—)
<i><math>V_{LOCI}(t) = 0</math> if <math>V_{GCM}(t) \leq \text{Threshold\_LOCI}</math></i>	<b>(14e)</b>
<i><math>V_{LOCI}(t) = \max(0, V_{GCM}(t) \times \text{ScaleFactor\_LOCI})</math> if <math>V_{GCM}(t) &gt; \text{Threshold\_LOCI}</math></i>	<b>(14f)</b>

## 11. How the Inverse CDF is Computed (with Tail Extension)

### New in V2.1 [NEW IN V2.1]

All quantile-based methods need to find the observed station value at a given probability  $p$ . This is done by sorting the observed data and interpolating. V2.1 adds linear tail extension: if a future GCM value has a probability  $p$  that falls outside the range of the observed historical data, the tool extrapolates beyond the minimum or maximum observed value using the slope at the tail. This prevents all extreme future values from being clamped to the historical maximum or minimum.

Let  $V_{Obs,sorted}$  be the observed data sorted from smallest to largest, with  $n$  values (index 0 to  $n-1$ ):

<i>Interior range</i> ( $1/n \leq p \leq (n-1)/n$ ):	(—)
$position = p \times (n - 1)$	(15a)
$lower\ index = \lfloor position \rfloor$ ; $upper\ index = lower + 1$ (capped at $n-1$ )	(15b)
$CDF^{-1}_{Obs}(p) = V_{Obs,sorted}[lower] + (position - lower) \times (V_{Obs,sorted}[upper] - V_{Obs,sorted}[lower])$	(15c)
<i>Below the observed range</i> ( $p < 1/n$ ):	(—)
$slope = (V_{Obs,sorted}[1] - V_{Obs,sorted}[0]) / (1/n)$	(16a)
$CDF^{-1}_{Obs}(p) = V_{Obs,sorted}[0] + slope \times (p - 1/n)$	(16b)
<i>Above the observed range</i> ( $p > (n-1)/n$ ):	(—)
$slope = (V_{Obs,sorted}[n-1] - V_{Obs,sorted}[n-2]) / (1/n)$	(17a)
$CDF^{-1}_{Obs}(p) = V_{Obs,sorted}[n-1] + slope \times (p - (n-1)/n)$	(17b)

Precipitation results are always clipped to  $\geq 0$  after this calculation.

## 12. Monthly Stratification (Optional)

New in V2.1 [NEW IN V2.1]

Monthly Stratification is activated by a checkbox in both the Evaluation tab and the Bias Correction tab.

### What Monthly Stratification does

Without Monthly Stratification (checkbox OFF): the bias-correction method uses all months of data together to build a single transfer function. This is called Global mode.

With Monthly Stratification (checkbox ON): the method is run 12 times — once for each calendar month. January data is corrected using only January observed and January GCM data; February is corrected using only February data; and so on.

Monthly Stratification is available for all six methods. The output is labelled with `_M`: `Delta_M`, `QM_M`, `EQM_M`, `QDM_M`, `DQM_M`, `SDM_M`.

Why use it: if the GCM bias varies by season (e.g. the model overestimates summer rain but underestimates winter rain), Monthly Stratification corrects each season independently.

### 12.1 How Each Month is Processed

1. The tool assigns every time step in the input data to its calendar month using the day-of-year.
2. For each month  $m$  (January through December), three monthly datasets are extracted: observed station data for month  $m$ , historical GCM data for month  $m$ , future GCM data for month  $m$ .
3. The selected bias-correction method is applied independently to each month's data.
4. The corrected values are placed back at their original time positions in the output.

### 12.2 Global vs Monthly Stratification: Delta Example

Mode	How $\delta$ is computed
Global (checkbox OFF)	One $\delta = \mu_{\text{Obs}} / \mu_{\text{GCM-Hist}}$ using all 12 months of data. Applied to all months equally.
Monthly (checkbox ON)	12 separate values: $\delta_{\text{Jan}} = \mu_{\text{Obs,Jan}} / \mu_{\text{GCM-Hist,Jan}}$ ; $\delta_{\text{Feb}} = \mu_{\text{Obs,Feb}} / \mu_{\text{GCM-Hist,Feb}}$ ; etc.
Impact	If the GCM has a 20% wet bias in summer but no bias in winter, Global mode over-corrects winter and correctly adjusts summer. Monthly Stratification applies the right correction to each season independently.

## 13. Evaluation Metrics

The Evaluation tab compares the bias-corrected output against the observed station data using these metrics. In all formulas:  $O_i$  = observed value,  $S_i$  = simulated (bias-corrected) value,  $\bar{O}$  = mean of observed,  $\bar{S}$  = mean of simulated,  $N$  = number of paired values.

Metric	Formula	Ideal Value	Reference
MAE	$(1/N) \sum  O_i - S_i $	0	Willmott (1981)
MBE	$(1/N) \sum (O_i - S_i)$	0	Standard
Pearson r	$\frac{\sum [(O_i - \bar{O})(S_i - \bar{S})]}{\sqrt{\sum (O_i - \bar{O})^2} \times \sqrt{\sum (S_i - \bar{S})^2}}$	1	Pearson (1895)
Spearman $\rho$	$1 - 6 \sum d_i^2 / [N(N^2 - 1)]$ ; $d_i = \text{rank}(O_i) - \text{rank}(S_i)$	1	Spearman (1904)
NSE	$1 - \sum (O_i - S_i)^2 / \sum (O_i - \bar{O})^2$	1	Nash & Sutcliffe (1970)
RMSE	$\sqrt{[(1/N) \sum (O_i - S_i)^2]}$	0	Standard
NRMSE	$RMSE / \bar{O}$	0	Standard
IoA (d)	$1 - \sum (O_i - S_i)^2 / \sum ( S_i - \bar{O}  +  O_i - \bar{O} )^2$	1	Willmott (1981)
KGE	$1 - \sqrt{[(r-1)^2 + (\alpha-1)^2 + (\beta-1)^2]}$	1	Gupta et al. (2009)
PBIAS	$100 \times \sum (O_i - S_i) / \sum O_i$ (%)	0%	Moriasi et al. (2007)
RSR	$RMSE / \sigma_{Obs}$	0	Moriasi et al. (2007)
Wet-Day Freq. Ratio	$(\# \text{ simulated wet days}) / (\# \text{ observed wet days})$	1	V2.1, Eq. 18

### 13.1 KGE Components

Symbol	Meaning for the User
$r$	Pearson correlation between observed and simulated values
$\alpha$	Variability ratio: standard deviation of simulated $\div$ standard deviation of observed
$\beta$	Mean ratio: mean of simulated $\div$ mean of observed
$KGE = 1$	Perfect agreement. Values above $-0.41$ are generally considered better than using the observed mean as a naive forecast.

## 13.2 Wet-Day Frequency Ratio

New in V2.1 [NEW IN V2.1]

<i>Number of observed wet days = <math>\#\{O_i &gt; \theta_{wet}\}</math> where <math>\theta_{wet} = 0.1 \text{ mm/day}</math></i>	<b>(18a)</b>
<i>Number of simulated wet days = <math>\#\{S_i &gt; \theta_{wet}\}</math></i>	<b>(18b)</b>
<i>Wet-Day Frequency Ratio = (simulated wet days) / (observed wet days)</i>	<b>(18c)</b>

A ratio of 1 means the bias-corrected data has the same number of rainy days as the observations. Only meaningful for precipitation data; displayed as N/A for temperature and other variables.

## 14. Wavelet Downscaling (Removed in V2.1)

**This feature has been removed. [REMOVED IN V2.1]**

In V2.0, users could combine any of the three methods (Delta, QM, EQM) with wavelet decomposition. The signal was split into approximation and detail components at multiple frequency levels using a Discrete Wavelet Transform (DWT), each component was bias-corrected separately, and the result was reconstructed using the Inverse DWT.

### Recommended alternatives in V2.1

QDM with Monthly Stratification (QDM\_M): preserves the GCM's projected change at every quantile and every season. Best choice for future projections where extremes are important.

SDM with Monthly Stratification (SDM\_M): corrects wet-day frequency and distributional changes simultaneously, with seasonal correction.

These methods provide better scientific results without the added complexity of wavelet decomposition.

## 15. Wet-Day Threshold

A fixed threshold is used throughout all precipitation calculations in V2.1:

$\theta_{wet} = 0.1 \text{ mm/day}$  (the minimum rainfall to count as a “wet day”)

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This threshold is used in: LOCI pre-processing, QM/EQM/QDM/DQM dry-day filtering, SDM wet-day ranking, and the Wet-Day Frequency Ratio metric. Any day with less than 0.1 mm of rainfall in the GCM output is set to 0.0 mm in the bias-corrected result.

## 16. References

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