Formulas of Drought Indices

1- *Meteorological drought*

A: Rain Based-drought indices (Salehnia et al., 2017):

- SPI (Standardized Precipitation Index, McKee et al. 1993, 1995
- DI (Deciles Index), Gibbs and Maher, 1967
- PN (Percent of Normal Index), Willeke et al. (1994)
- CZI (China-Z Index), Wu et al. (2001)
- MCZI (Modified CZI), Wu et al. (2001)
- EDI (Effective drought Index), Byun and Wilhite (1999)
- RAI (Rainfall Anomaly Index), van Rooy (1965)
- ZSI (Z-score Index), Palmer (1965)

B: Other meteorological drought indices:

- PDSI (Palmer Drought Severity Index), Palmer (1965) and Dehghan et al., 2020
- PHDI (Palmer Hydrological Drought Index), Palmer (1965)
- SPEI (Standardized Precipitation Evapotranspiration Index), Vicente-Serrano et al., 2010 and Salehnia et al., 2020
- RDI (Reconnaissance Drought Index), Tsakiris and Vangelis, 2005.

2- *Agricultural drought indices*

- ARI (Agricultural Rainfall Index), Nieuwolt, 1981
- SMDI (Soil Moisture Deficit Index), Narasimhan and Srinivasan, 2005
- ETDI (Evapotranspiration Deficit Index), Narasimhan and Srinivasan, 2005

3- Hydrological drought indices

- SWSI (Surface Water Supply Index), Garen, 1993
- SDI (Streamflow Drought Index), Nalbantis and Tsakiris, 2009

1.1 SPI (Standardized Precipitation Index)

The SPI stands as the predominant drought index on a global scale, utilized for monitoring and comprehensive analysis, as highlighted by Karabulut (2015). Widely acknowledged, the SPI serves as a crucial instrument for delineating meteorological drought patterns, as evidenced by the works of Hayes et al. (1999) and Deo (2011). The pioneering work of McKee et al. (1993, 1995) encompassed the establishment of SPI across multiple temporal scales (1, 3, 6, 12, 24, and 48 months). The resultant SPI values span from +2.0 to −2.0. Given the possibility of precipitation data conforming to a gamma distribution, the calculation of SPI entails the utilization of the probability density function inherent to the gamma distribution:

$$g(x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{\frac{-x}{\beta}} \quad for \quad x \quad > 0$$

In this context, the gamma function is denoted by Γ, where x represents the quantity of precipitation (x > 0), α (> 0) stands as the shape parameter, and β (> 0) represents the scale parameter. For further insights, refer to the comprehensive studies conducted by Edwards and McKee (1997) as well as Dogan et al. (2012).

1.2 PN (Percent of Normal Precipitation Index)

The PN index, delineated by Willeke et al. (1994), represents the percentage of normal precipitation attributed to a particular location. This metric, PN, holds the versatility to be computed over various temporal scales such as monthly, seasonal, and annual periods. Notably, when applied to a singular geographic region or specific season, PN has demonstrated its efficacy in elucidating drought conditions, as demonstrated by the research of Hayes (2003). The calculation of PN is as follows:

$$PN = {}^{P_i}/_{P} \times 100$$

Here, P_i denotes the precipitation during the time increment labeled as "i," while P represents the mean normal precipitation for the duration of the study period.

1.3 DI (Deciles Index)

The deciles index (DI) was defined by Gibbs and Maher (1967). Monthly historical precipitation data are sorted from lowest to highest and divided into ten equal categories or deciles, so a given month's precipitation is placed into historical context by decile.

To compute the deciles index in DMAP, we first calculate the deciles for each month and subsequently identify any specific month-year in which the deciles meet certain criteria.

$$DEC_n = Per(P_m, n)$$

$$DI_{i,j} = n \quad if \quad DEC_{n-1} < P_{i,j} \leq DEC_n$$

The term DEC_n represents the deciles associated with the value of n, *where n takes on values such as 10, 20, ..., 90. Here, Per signifies the percentile function. The P_m corresponds to the rainfall observed in month m.*

1.4 EDI (Effective Drought Index)

The Effective Drought Index (EDI) is computed at a daily time step and follows a standardization process akin to SPI. Byun and Wilhite (1999) pioneered the development of EDI, aiming to address certain shortcomings observed in other indices like SPI. The EDI's numerical range spans from -2.5 to 2.5. When the EDI falls within the range of -1 to 1, it signifies near-normal conditions, whereas values below -2 indicate severe drought. The calculation of EDI is accomplished through the application of the subsequent equation:

$$EP_i = \sum_{DS}^{n=1} \left[\left(\sum_{n}^{m=1} P_{(i-m)}\right) / n \right]$$

$$DEP_i = EP_i - MEP$$

$$EDI_i = {DEP_i}/{SD(DEP)}$$

Where DS represents the count of days used for summing precipitation in the calculation of drought severity (usually 365 or 15), SD stands for standard deviation, and MEP stands for the mean of EP. The DMAP calculates EDI on daily scale then you can use Monthly, Yearly, or seasonally to convert daily EDI to this time scales.

1.5 CZI (China-Z Index)

In 1995, the National Climate Center of China introduced the CZI as a substitute for the SPI (Ju et al., 1997) specifically designed for situations where mean precipitation adheres to the Pearson Type III distribution. The CZI is computed as follows:

$$CZI_{ij} = \frac{6}{C_{si}} \times \left(\frac{C_{si}}{2} \times \phi_{ij} + 1\right)^{1/3} - \frac{6}{C_{si}} + \frac{C_{si}}{6}$$

$$C_{si} = \frac{\sum_{j=1}^{n}(X_{ij} - \bar{X}_i)^3}{n \times \sigma_i^3}$$

$$\varphi_{ij} = \frac{X_{ij} - \bar{X}_i}{\sigma_i}$$

$$\sigma_i = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(X_{ij} - \bar{X}_i)^2}$$

where *i* is the time scale of interest, j is the current month, C_{si} is the coefficient of skewness and φ_{ij} is the standardized variation. Further details can be found in Wu et al., 2001.

1.6 MCZI (Modified CZI)

The adjusted MCZI employs the aforementioned formula but replaces the mean precipitation with the median precipitation.

1.7 ZSI (Z-Score Index)

The Z-Score is sometimes mistaken for SPI. However, it bears a closer resemblance to CZI, although it doesn't necessitate fitting precipitation data into either Gamma or Pearson Type III distributions:

$$ZSI = \frac{P_i - \bar{P}}{SD}$$

Here, P represents the average monthly precipitation (mm), P_i stands for the precipitation in a particular month, and SD denotes the standard deviation across any time scale.

1.8 RAI (Rainfall Anomaly Index)

The RAI involves two phases: positive anomalies and negative anomalies. Initially, precipitation data are sorted in descending order. The top ten and bottom ten data points are then averaged separately to create these phases. The positive and negative anomalies are computed using equations 6 and 7, respectively:

$$RAI = 3 \times \left[\frac{(p - \bar{p})}{(\bar{m} - \bar{p})}\right]$$

$$RAI = -3 \times \left[\frac{(p - \bar{p})}{(\bar{m} - \bar{p})}\right]$$

In this context, RAI stands for Rainfall Anomaly Index. Here, p represents the actual precipitation (mm) for each year, $\overline{p}$ is the long-term average precipitation, and $\overline{m}$ is the mean of the top ten values of p for the positive anomaly, as well as the mean of the bottom ten values of p for the negative anomaly.

Reference **for 1.1 up to 1.8**: Estimation of meteorological drought indices based on AgMERRA precipitation data and station-observed precipitation data

1.9 KBDI (Keetch-Byram Drought Index)

Calculating the index requires data for daily maximum temperature and precipitation. Following that, the KBDI was computed using the subsequent methodology:

$$DF = \frac{[800 - KBDI_{t-1}] \times [0.968 \times Exp(0.875T_{max} + 1.5552) - 8.30]}{1 + 10.88 \times Exp(-0.001736R)} \times 10^{-3}$$

$$KBDI_t = \begin{cases} KBDI_{t-1} & if\ P = 0\ and\ Tmax \leq 6.78\ C \\ KBDI_{t-1} + DF & if\ P = 0\ and\ Tmax > 6.78\ C \\ KBDI_{t-1} + DF & if\ P > 0\ and\ \sum P \leq 5.1\ mm \\ KBDI_j + DF & if\ P > 0\ and\ \sum P > 5.1\ mm \end{cases}$$

$$KBDI_j = KBDI_{t-1} - 39.37 \times \sum p$$

The drought factor (DF) on a given day in the metric system, as presented by Janis et al. in 2002, is calculated based on several factors. Specifically, it depends on the daily maximum temperature ($Tmax_t$) in degrees Celsius, the average annual rainfall (R) in centimeters for the region, the Keetch-Byram drought index for the previous day ($KBDI_{t-1}$), and the daily precipitation (P_t) in millimeters (Janis et al. 2002). The calculation involves intricate interplays between these variables. Notably, the initialization of the Keetch-Byram drought index (KBDI) is of paramount importance. Traditionally, this initialization occurs after a series of consecutive rainy days when soil saturation is attained. However, it's important to acknowledge that the field capacity of arable land, which signifies its ability to absorb water, can differ significantly due to variations in soil composition and changes in vegetation.

Referenc: Predictive value of Keetch-Byram Drought Index for cereal yields in a semi-arid environment

1.10 PDSI

1.11 PHDI

Calculating these two indices involves a substantial and intricate process. It's advised to consult the original paper (Meteorological drought Palmer 1965 Washington DC) for detailed information. Additionally, our calculation of the indices is directly based on this paper: A tool for calculating the Palmer drought indices

1.12 SPEI

In the computation of SPEI, we employed both the classical approximation by Abramowitz and Stegun (1965) and the method proposed by Vicente-Serrano et al. (2009). The SPEI, accumulated over various time scales, is founded upon a monthly climatic water balance derived from the difference between precipitation and

Evapotranspiration (ETO). This balance is then refined through the application of a three-parameter log-logistic distribution. The accumulation of deficit or surplus in the climatic water balance (D) at different time scales is ascertained by evaluating the dissimilarity between daily precipitation (P) and ETO for a given day "i":

$$D_i = P_i - ETo_i$$

The probability density function of a three-parameter log-logistic distributed variable is expressed as below:

$$f(x) = \frac{\beta}{\alpha}\left(\frac{x-\gamma}{\alpha}\right)^{\beta-1} \times \left[1 + \left(\frac{x-\gamma}{\alpha}\right)^{\beta}\right]^{-2}$$

Here, α, β, and γ represent the scale, shape, and origin parameters, respectively, for D values within the range (γ > D < ∞). The parameters for the Pearson III distribution can be acquired using the following method:

$$\beta = \frac{2w_1 - w_0}{6w_1 - w_0 - 6w_2}$$

$$\alpha = \frac{(w_0 - 2w_1)\beta}{\Gamma(1 + {}^{1}/_{\beta})\Gamma(1 - {}^{1}/_{\beta})}$$

$$\gamma = w_0 - \alpha\Gamma\left(\frac{1+1}{\beta}\right)\Gamma\left(\frac{1-1}{\beta}\right)$$

Here, Γ(β) denotes the gamma function of β. The probability distribution function of the series "D" is expressed as follows:

$$F(x) = \left[1 + \left(\frac{\alpha}{x-\gamma}\right)^{\beta}\right]^{-1}$$

In the final stage, by utilizing the value of F(x), the SPEI can be determined as the standardized value of F(x). The calculation of the SPEI equation is as follows:

$$\mathrm{SPEI} = \mathrm{W} - \frac{C_0 + C_1 W + C_2 W^2}{1 + d_1 W + d_2 W^2 + d_3 W^3}$$

Here, for cases where P ≤ 0.5, W is defined as the square root of -2 times the natural logarithm of P, where P represents the probability of surpassing a specific value of "D." The constants involved are as follows: C0 = 2.515517, C1 = 0.802853, C2 = 0.010328, d1 = 1.432788, d2 = 0.189269, and d3 = 0.001308.

Reference: Rainfed wheat (Triticum aestivum L.) yield prediction using economical, meteorological, and drought indicators through pooled panel data and statistical downscaling

1.13 RDI

RDI (Reconnaissance Drought Index), developed by Tsakiris and Vangelis in 2005, compares two cumulative measures: precipitation and potential evapotranspiration. For a given month "m", this calculation for a yearly index is represented by the following equation:

$$D_m = \frac{\sum_{i=Jan}^{Dec} P_i}{\sum_{i=Jan}^{Dec} PET_i}$$

The Reconnaissance Drought Index (RDI) is characterized by two variations: the normalized RDI and the standardized RDI. The normalized RDI serves as the default option within DMAP.

The Normalized Reconnaissance Drought Index (RDI) is computed using the following equation:

$$RDI_m = \frac{D_m}{\bar{D}} - 1$$

The $\overline{D}$ is average of D for all years. The Standardized Reconnaissance Drought Index (RDI) is computed using the following equation:

$$RDI_m = \frac{D_m - \overline{D}}{\sigma}$$

The $\overline{D}$ is average of D for all years and σ is standard deviation of D for all years.

Reference: Regional Drought Assessment Based on the Reconnaissance Drought Index (RDI)

1.14 SMDI

For the computation of SMDI, it is necessary to employ the long-term median, maximum, and minimum monthly (or seasonal or yearly) soil water values. The following formulas are utilized for this purpose:

$$SD_{i,j} = \begin{cases} \dfrac{SW_{i,j} - MSW_j}{MSW_j - minSW_j} \times 100 & if\ SW_{i,j} \leq MSW_j \\ \dfrac{SW_{i,j} - MSW_j}{maxSW_j - MSW_j} \times 100 & if\ SW_{i,j} > MSW_j \end{cases}$$

$$SMDI_j = 0.5 \times SMDI_{j-1} + \frac{SD_j}{50}$$

The SMDI values will range from -4 to 4 for comparison with PDSI values.

Here, MWS_j represents the long-term median of water stress for month j, $maxMWS_j$ stands for the long-term maximum water stress of month j, $minWS_j$ signifies the long-term minimum water stress of month j, and WS corresponds to the monthly water stress. The subscripts i and j are employed for years and months respectively. The water stress anomaly varies from -100 to +100 for each month, denoting exceedingly dry to exceptionally wet conditions.

Reference: Development and evaluation of Soil Moisture Deficit Index (SMDI) and Evapotranspiration Deficit Index (ETDI) for agricultural drought monitoring

1.15 ARI

ARI serves as a dependable indicator of monthly water equilibrium, specifically designed for assessing agricultural droughts. The calculation of ARI follows this methodology:

$$ARI = \frac{P}{PET}$$

Here, P represents the monthly precipitation (mm/month), and ET0 signifies the reference evapotranspiration (mm/month). The presence of drought within a specific month is denoted by ARI values of 40. A range of ARI values between 40 and 200 indicates favorable conditions for vegetation growth and agricultural productivity. An ARI exceeding 200 for a month signifies a period of elevated moisture, indicating wet conditions.

Reference: Understanding Dry and Wet Conditions in the Vietnamese Mekong Delta Using Multiple Drought Indices: A Case Study in Ca Mau Province

1.16 ETDI

The ETDI is computed similarly to the SMDI, yet it relies on the deviation of water stress from its long-term mean. Here, the monthly water stress is established through the interplay of potential and actual evapotranspiration.

$$WS = \frac{PET - AET}{PET}$$

$$WSA_{i,j} = \begin{cases} \dfrac{WS_{i,j} - MWS_j}{MWS_j - minWS_j} \times 100 & if\ WS_{i,j} \le MWS_j \\ \dfrac{WS_{i,j} - MWS_j}{maxWS_j - MWS_j} \times 100 & if\ WS_{i,j} > MWS_j \end{cases}$$

$$ETDI_j = 0.5 \times ETDI_{j-1} + \frac{WSA_j}{50}$$

The ETDI values will range from -4 to 4 for comparison with PDSI values.

Reference: Development and evaluation of Soil Moisture Deficit Index (SMDI) and Evapotranspiration Deficit Index (ETDI) for agricultural drought monitoring

1.17 SWSI

A prevalent hydrological drought index consists of a series of practical individual measures, known as the Surface Water Drought Severity Index (SWSI). The formula used to compute this index, as per Garen's investigations, can be described as follows:

$$SWSI_{i,j} = \frac{100 \times P_{i,j} - 50}{12}$$

$$P_{i,j} = GammaCDF(StreamFlow_{i,j})$$

The GammaCDF represents the cumulative distribution function of the gamma distribution fitted onto month j across all years. In this context, $StreamFlow_{i,j}$ denotes the streamflow during month j of the year i.

Reference: Comparing surface water supply index and streamflow drought index for hydrological drought analysis in Ethiopia

1.18 SDI

The Streamflow Drought Index can be computed using the following formula when applied to monthly data. However, if you provide daily data, the tool will automatically aggregate it into monthly values before performing the calculation.

$$V_{i,k} = \sum_{j=1}^{3K} SF_{i,j} \qquad i = 1,2,..,endYear \quad j = 1,2,..,12 \quad k = 1,2,3,4$$

$$SDI_{i,k} = \frac{V_{i,k} - \bar{V}_k}{SD_k}$$

The premise is that a time series of monthly streamflow volumes is available, denoted as $SF_{i,j}$. Here, i indicates the hydrological year, and j represents the month within that hydrological year (where j = 1 corresponds to October and j = 12 corresponds to September). In this context, $\bar{V}_k$ and SD_k stand for the mean and standard deviation of cumulative streamflow volumes observed during the reference period k, established over a substantial time span. Within the framework of DMAP, the resulting output will encompass these four categories: Oct-Dec, Oct-Mar, Oct-Jun, and Oct-Sep (Yearly).

Reference: Assessment of Hydrological Drought Revisited