Formulas in SD-GCM

In the SD GCM tool, users have access to three statistical downscaling (SD) methods, which include the following models:

1. The Delta method

2. The Quantile Mapping (QM) method (Panofsky and Briar, 1968)

3. The Empirical Quantile Mapping (EQM) method (Boe et al., 2007)

A- Delta Method:

The Delta method is a statistical downscaling technique used in climate science and hydrology to estimate local or regional climate variables, such as precipitation or temperature, from the output of global climate models (GCMs) or regional climate models (RCMs). This method is relatively simple and relies on the concept of change or delta between the historical observed climate data and the modeled climate data.

Here's a basic description of how the Delta method works:

- Calculate the difference or delta between the GCM/RCM model output and the observed historical data for a specific time period (e.g., monthly or annually).

These deltas represent the modeled change in climate conditions compared to historical conditions.

- To downscale future climate projections, apply the calculated deltas to the future GCM/RCM model data. Essentially, you are adding or subtracting the deltas to the model output to adjust it to conditions more similar to historical observations.

The Delta method is a useful tool for obtaining localized climate projections when more complex and computationally intensive downscaling methods are not feasible, but it should be used with an awareness of its limitations and assumptions.

As presented in Eq. 1 and Eq. 2 the precipitation and temperature of GCM data are downscaled

$$P_{SD-Fut}^{Delta} = P_{GCM-RCP/SSP} \times \frac{\bar{P}_{Obs}}{\bar{P}_{GCMhist.}} \quad (1)$$

$$T_{SD-Fut}^{Delta} = T_{GCM-RCP/SSP} + (\bar{T}_{Obs} - \bar{T}_{GCMhist}) \quad (2)$$

$$P_{SD-Eval}^{Delta} = P_{GCMhist} \times \frac{\bar{P}_{Obs}}{\bar{P}_{GCMhist.}} \quad (1\text{-}1)$$

$$T_{SD-Eval}^{Delta} = T_{GCMhist} + (\bar{T}_{Obs} - \bar{T}_{GCMhist}) \quad (2\text{-}1)$$

In this equation, we introduce and use various data components as follows:

1. Future Period Downscaled Data: P_{SD-Fut}^{Delta} and T_{SD-Fut}^{Delta} denote downscaled precipitation and temperature data, respectively, specifically for the future period.

2. Evaluation Period Downscaled Data: $P_{SD-Eval}^{Delta}$ and $T_{SD-Eval}^{Delta}$ represent downscaled precipitation and temperature data, respectively, designed for the evaluation period.

3. GCM Scenario Data: $P_{GCM-RCP/SSP}$ and $T_{GCM-RCP/SSP}$ represent data derived from Global Climate Models (GCMs) under various Shared Socioeconomic Pathways (SSPs) or Representative Concentration Pathways (RCPs). These datasets serve as scenario-based inputs.

4. GCM Historical Data: $P_{GCGhist}$ and $T_{GCMhist}$ represent historical data obtained from Global Climate Models (GCMs), specifically from each individual model.

5. Average Observed Mean Data: $\bar{P}_{Obs}$ and $\bar{T}_{Obs}$ signifies the average observed precipitation and temperature over a specified period.

6. Historical GCM Mean Data: $\bar{P}_{GCMhist}$ and $\bar{T}_{GCMhist.}$ refers to the GCM's mean simulation data for historical precipitation records.

B- Quantile Mapping (QM)

Quantile Mapping (QM) is a statistical downscaling method commonly used in climate science and meteorology to bridge the gap between coarse-resolution climate model output and finer-scale, local climate data. This method is particularly useful when you want to make climate model projections more relevant and accurate for specific regions or locations.

You calculate the cumulative distribution functions (CDFs) for both the observed (high-resolution) data and the climate model (coarse-resolution) data. The CDF essentially shows the probability of observing a particular value or less. For each value in the climate model data, you find the corresponding quantile from the observed data's CDF. This step involves matching the probabilities from the model to the observed data. After mapping the quantiles, you adjust the climate model data values based on the quantile mapping. The adjusted climate model data now closely resemble the statistical characteristics of the observed data, including the distribution of values and their variability. Quantile Mapping helps in addressing biases and discrepancies between climate model outputs and observed data. By preserving the statistical properties of the observed data, it provides more reliable projections of future climate conditions at finer spatial and temporal scales.

Some advantages of Quantile Mapping include its simplicity and ability to correct for both mean biases and variability biases. However, it's essential to note that this method assumes a stationary relationship between the historical and future climate, which may not always hold true in the face of significant climate change. In such cases, more complex downscaling methods or additional considerations may be necessary.

This concept is calculated according to Equation 3, applicable to precipitation data (or any other variable).

$$P_t^{Eval} = InvCDF_{P_t}^{obs}\left(CDF_{P_t}^{Hist}\left(P_{t,Hist}\right)\right) \quad (3\text{-}1)$$

$$P_t^{Predict} = InvCDF_{P_t}^{obs}\left(CDF_{P_t}^{Hist}\left(P_{t,RCP/SSP}\right)\right) \quad (3\text{-}2)$$

Where $InvCDF_{P_t}^{obs}$ represents the inverse cumulative distribution function (CDF) based on observational (or station) data.

$CDF_{P_t}^{Hist}$ represents the cumulative distribution function (CDF) based on historical period of GCMs model.

$P_{t,RCP/SSP}$ represent each value of precipitation time series in RCPs/SSPs scenarios of GCMs model.

$P_{t,Hist}$ represent each value of precipitation time series in historical period of GCMs model.

The formula provided above remains consistent for any variable; only the distribution utilized can vary. In SD-GCM, we employ a Gamma distribution for the Multiplicative Correction Factor and Normal distributions for the Additive Correction Factor.

C- Empirical Quantile Mapping (EQM)

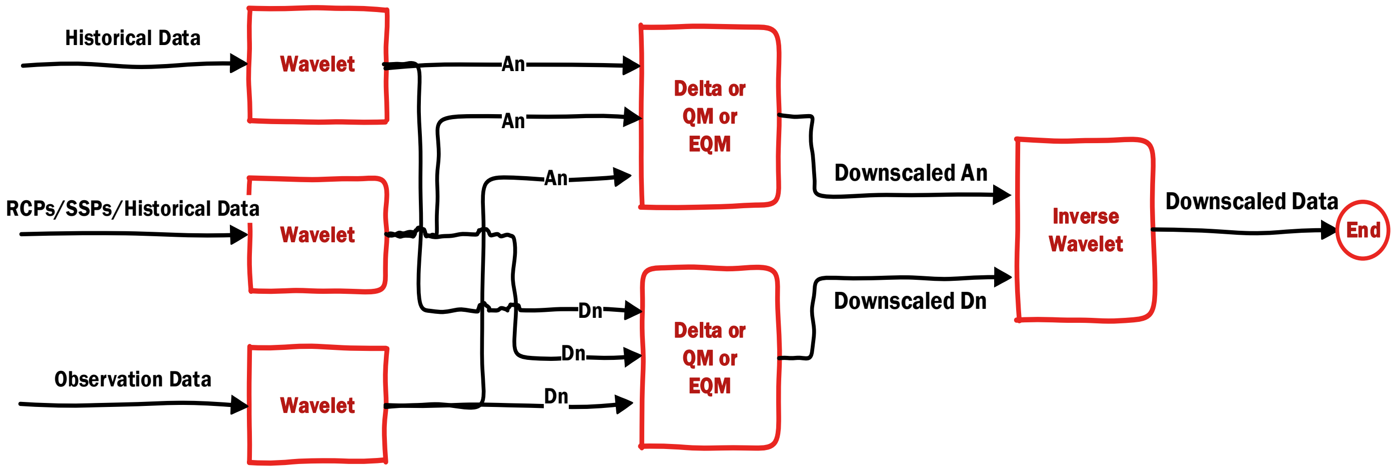
Quantile Mapping (QM) and Empirical Quantile Mapping (EQM) are both statistical downscaling techniques used in climate science and meteorology and the formula is similar, but they have distinct differences in their approaches:

To use QM, we should select a distribution for data like Normal, Gamma... distribution but EQM is a specific implementation of quantile mapping that relies solely on empirical data without assuming any specific probability distribution for the data. The key feature is that EQM directly maps quantiles from the model data to the quantiles of the observed data, without making any distributional assumptions. EQM is nonparametric in nature. It doesn't rely on fitting a particular probability distribution to the data but instead uses the data itself to perform the mapping.

In summary, while both Quantile Mapping (QM) and Empirical Quantile Mapping (EQM) are methods for adjusting climate model data to match observed data, EQM is a specific implementation of QM that is nonparametric and relies entirely on empirical data for the mapping, making it a simpler and more data-driven approach. QM, on the other hand, is a broader framework that can involve different types of quantile mapping, including parametric methods that assume specific probability distributions for the data. The choice between QM and EQM depends on the specific research objectives and the characteristics of the data being used.

D- Compose with Wavelet

We have recently added a new icon to the tool, enabling you to integrate wavelet analysis with the existing methods. To gain a deeper understanding of how this feature operates, please consult the flowchart provided below:



As depicted in the flowchart, wavelet transformation can be integrated with any of the three methods mentioned above. Given the variability in wavelet transform levels and numbers, you have the option to check a broad array of techniques by combining them with the three exponential scaling methods.

Wavelet Parameters

In the context of wavelet analysis, "wavelet type," "number," and "level" refer to key parameters that determine how wavelet transformations are applied to a signal or data set:

1. Wavelet Type:

- The "wavelet type" specifies the shape of the wavelet function that will be used for the analysis. Different wavelet types are designed to capture different types of patterns or features in the data. Common wavelet types include Daubechies (db), Symlet(sym), and Coiflet(coif), among others. The choice of wavelet type depends on the characteristics of the data and the specific goals of the analysis.

2. Number of Wavelets:

- The "number of wavelets" typically refers to the number of wavelet functions used in a wavelet transform. In discrete wavelet transforms, this often means how many scaling and wavelet functions are employed at each level of the transformation. For example, a common choice is to use two wavelets: one for approximation (scaling) and one for detail (wavelet) at each level. However, in some cases, more wavelets may be used to capture finer details or different frequency components.

3. Level of Decomposition:

- The "level of decomposition" or "number of decomposition levels" indicates how many times the data will be iteratively transformed using wavelets. Each level of decomposition divides the data into different frequency components or scales. A higher level of decomposition provides more detailed information about the data's frequency content but may also result in a larger number of transformed coefficients.

These parameters are essential for customizing wavelet analysis to suit the specific characteristics of the data and the goals of the analysis. The choice of wavelet type, number, and level can have a significant impact on the results and the ability to extract relevant information from the data. It's often necessary to experiment with different parameter settings to find the most suitable configuration for a particular analysis.